- A matrix $A \in \mathbb{R}^{n \times n}$ represents a linear transformation:

$$A : \mathbb{R}^n \to \mathbb{R}^n$$

  where it takes any vector $x \in \mathbb{R}^n$ into a new vector $Ax \in \mathbb{R}^n$.

- An eigenvector is a nonzero direction such that any vector parallel to it is mapped by a linear transformation to a new vector scaled by the corresponding eigenvalue λ, in the same direction if $\lambda > 0$ and opposite direction if $\lambda < 0$.

  Specifically, for a vector $x \in \mathbb{R}^n$, if $x = kv$,

$$Ax = A(kv) = k(Av) = k(\lambda v) = \lambda(kv)$$

- For any vector, a linear transformation (whose matrix has n eigenvectors and corresponding eigenvalues) maps it to a new vector as follows:
  - **Decomposition**: The vector is expressed as a linear combination of the eigenvectors, with each term being a component along an eigenvector direction.
  - **Transformation**: Each component is scaled by its corresponding eigenvalue, without changing its direction.
  - **Summation**: The transformed vector is obtained by summing these scaled components.

  Specifically, for a vector $x \in \mathbb{R}^n$, if $x = \sum_{i=1}^{n} k_i v_i$,

$$Ax = A \sum_{i=1}^{n} k_i v_i = \sum_{i=1}^{n} k_i (A v_i) = \sum_{i=1}^{n} k_i (\lambda_i v_i) = \sum_{i=1}^{n} \lambda_i (k_i\, v_i)$$